

# Superluminal Neutrinos in the Minimal Standard Model Extension

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The measurement of the neutrino velocity with the OPERA detector in the CNGS beam shows unexpected indication, that the muon neutrino velocity,  $v_\nu$ , exceeds the velocity of light in the vacuum,  $c$ , which is obviously in contradiction with the most basic hypothesis of modern physics. Within the framework of minimal Standard Model Extension, we discuss the modified dispersion relation and consequently the velocity-energy relation of muon neutrinos. The simplified models are fitted to the OPERA data, Fermilab experiment and MINOS data. We find that minimal Standard Model Extension can describe these long baseline superluminal neutrinos to a good accuracy. For the well-known tension between the OPERA measurement and the Supernova 1987A neutrino observation, we discuss two ways out of the contradiction.

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## I. SUPERLUMINAL NEUTRINOS IN OPERA, FERMILAB AND MINOS EXPERIMENTS

Recently the OPERA neutrino experiment at the underground Gran Sasso Laboratory reported their measurement of the velocity of neutrinos from the CERN CNGS beam over a baseline of about 730 km [1]. Compared to the time taken for neutrinos traveling at the speed of light in vacuum, an early arrival time of  $(60.7 \pm 6.9 \text{ (stat.)} \pm 7.4 \text{ (sys.) ns}$  was measured. This anomaly corresponds to a relative difference of the muon neutrino velocity with respect to the speed of light

$$\frac{v_\nu - c}{c} = (2.48 \pm 0.28 \text{ (stat.)} \pm 0.30 \text{ (sys.)}) \times 10^{-5}, \quad (1)$$

at a significance of  $6\sigma$ . The OPERA result has already inspired many papers with various discussions and models [2]. This is largely because there are previously some supportive results from other collaborations. The first direct measurement of neutrino velocity has been performed at Fermilab long ago [3, 4]. Based on 9,800 events, measurements of the velocity of muon neutrinos with energy ranging from 30 GeV to 200 GeV gave

$$|\beta_\nu - 1| < 10^{-5}, \quad (2)$$

where  $\beta_\nu \equiv v_\nu/c$ . A few years ago, by using the NuMI neutrino beam, the MINOS collaboration analyzed a total of 473 far detector neutrino events with an average energy  $\sim 3$  GeV [5]. They reported a shift with respect to the expected time of flight of

$$\delta_t = -126 \pm 32 \text{ (stat.)} \pm 64 \text{ (sys.) ns} \quad (3)$$

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which corresponds to a constraint on the muon neutrino velocity,

$$\frac{v_\nu - c}{c} = (5.1 \pm 2.9) \times 10^{-5} \quad (4)$$

at 68% confidence level. This  $1.8\sigma$  signal was considered consistent with zero, therefore it does not provide a strong evidence of Lorentz violation effects. However, with the new measurement of OPERA detector, it is surprising to notice that the MINOS results and the OPERA results are compatible.

Although these measurements have been largely debated, the neutrino velocity anomaly appears to be a strong challenge to the well-known fact in special relativity that no physical signal travels faster than the light. Special relativity, or Lorentz symmetry, is the profound symmetry of space-time and has been incorporated into the two cornerstones of modern physics: General Relativity and Quantum Field Theory. However, the possible Lorentz symmetry violation (LV) effects are sought for decades from various species of the standard model, motivated by the unknown underlying theory of quantum gravity together with various phenomenological applications [6–9]. This can happen in many alternative theories, e.g., the doubly special relativity (DSR) [10, 11], torsion in general relativity [12, 13], and large extra-dimensions [14, 15]. In this paper, we will work in an effective field theory framework, the Standard-Model Extension (SME), in which Lorentz violation terms constructed with Standard Model (SM) fields and controlling coefficients are added to the usual SM lagrangian [16, 17]. The origins for such LV operators are suggested in many ways, of which spontaneous Lorentz symmetry breaking proposed first in string theory is widely recognized [18, 19]. There is also a recent proposal to derive some supplementary LV terms from standard model with a basic consideration of the physical invariance with respect to the mathematical background manifolds [20, 21], and such a framework has been applied to discuss the Lorentz violation effects for the cases of Dirac particles [20], photons [21–23], and neutrinos [24], in which the superluminal neutrinos as a signal of Lorentz violation was suggested. In fact, the possibilities of superluminal neutrinos were proposed with an earlier version of SME [25] and with extra-dimensions [15]. Here we do not get into such discussions on theoretical details, but just analyze the experiments of superluminal neutrinos with the effective lagrangian of SME given in the next section.

## II. SME IN NEUTRINO SECTOR AND THE DISPERSION RELATION

The SME lagrangian in neutrino sector takes the form [17, 26, 27]

$$\mathcal{L} = \frac{1}{2} i \bar{\nu}_A \gamma^\mu \overleftrightarrow{D}_\mu \nu_B \delta_{AB} + \frac{1}{2} i c_{AB}^{\mu\nu} \bar{\nu}_A \gamma_\mu \overleftrightarrow{D}_\nu \nu_B - a_{AB}^\mu \bar{\nu}_A \gamma_\mu \nu_B + \dots, \quad (5)$$

where  $c_{AB}^{\mu\nu}$  and  $a_{AB}^\mu$  are Lorentz violation coefficients resulting from tensor vacuum expectation values in the underlying theory, the subscripts  $A, B$  are flavor indices, and the ellipsis denotes the non-renormalizable operators (eliminated in the minimal SME). The first term in Eq. (5) is exactly the SM operator, the second and third terms (CPT-even and CPT-odd respectively) describe the contribution from Lorentz violation. For simplicity, we neglect the effects due to interactions between neutrinos and matters in which the neutrino beam propagates, and thus replace  $D_\mu$  with  $\partial_\mu$  in Eq. (5). Then after a simple transformation, we arrive at

$$\mathcal{L} = i \bar{\nu}_A \gamma^\mu \partial_\mu \nu_B \delta_{AB} + i c_{AB}^{\mu\nu} \bar{\nu}_A \gamma_\mu \partial_\nu \nu_B - a_{AB}^\mu \bar{\nu}_A \gamma_\mu \nu_B. \quad (6)$$

With Eq. (6), we can easily get the Euler-Lagrange equation for neutrinos as

$$(i \gamma^\mu \partial_\mu \delta_{AB} + c_{AB}^{\mu\nu} \gamma_\mu \partial_\nu - a_{AB}^\mu \gamma_\mu) \nu_B = 0. \quad (7)$$

One would find that the neutrino mass terms are missing here. This is because such terms contribute to the neutrino velocity in the form of  $(\frac{m}{E})^2$ , totally negligible when GeV neutrinos are discussed. Following the procedure presented in Appendix. A of Ref. [28], we arrive at the effective hamiltonian as

$$(H_{\text{eff}})_{AB} = \begin{pmatrix} |\vec{p}| \delta_{AB} + a_{AB}^{\mu} \frac{p_{\mu}}{|\vec{p}|} - c_{AB}^{\mu\nu} \frac{p_{\mu} p_{\nu}}{|\vec{p}|} & 0 \\ 0 & |\vec{p}| \delta_{AB} - a_{AB}^{\mu} \frac{p_{\mu}}{|\vec{p}|} - c_{AB}^{\mu\nu} \frac{p_{\mu} p_{\nu}}{|\vec{p}|} \end{pmatrix}, \quad (8)$$

which is a  $6 \times 6$  matrix when three generations of neutrinos are considered, and the up-diagonal matrix denotes the hamiltonian for neutrinos while the down-diagonal matrix denotes the hamiltonian for anti-neutrinos. By diagonalizing this hamiltonian, one can get the eigenenergies, the mixing matrix and consequently oscillation probabilities of neutrinos. However, we are just interested in the velocity or the dispersion relation of neutrinos, thus the oscillation effects can be neglected in a first approximation. Therefore, the model is simplified by including only one flavor, i.e.,  $\nu_{\mu}$ , so Eq. (8) reduces to the dispersion relation of the muon neutrinos as

$$E = |\vec{p}| + \frac{1}{|\vec{p}|} (a^{\mu} p_{\mu} - c^{\mu\nu} p_{\mu} p_{\nu}). \quad (9)$$

With this dispersion relation, we could deduce the energy dependence of the neutrino velocity. By comparing with experiment results presented in the previous section, we can also fit the Lorentz violation coefficients.

### III. FITS TO MUON NEUTRINO VELOCITY MEASUREMENTS

We hold the same criterion when selecting data with that of Ref. [29]. Data are collected from the measurements of the muon neutrino velocity of Fermilab [3, 4], MINOS [5], and OPERA [1] (even data of muon anti-neutrinos from Fermilab are abandoned to keep things as clean as possible). For Fermilab data, as noticed in their paper, there might be a potential bias  $b = b_{0-\sigma_b}^{+\sigma_b} = 5_{-1}^{+2} \times 10^{-5}$  to the measured  $v_{\nu} - c$ . Hence before using them, we corrected the bias, and the error bars are squaredly summed, i.e.,  $\sigma_{\text{new}}^2 = \sigma_{\text{old}}^2 + \sigma_b^2$ . The bias-corrected data are listed in Table I. For MINOS results, though the energy spectrum for neutrinos has a long high-energy tail extending to 120 GeV, we pick the peak value 3 GeV, which might induce some unknown bias, but while the error bar is large for this data point, its contribution to fitting is relatively small. For OPERA data, for our purpose, we only use the  $\nu_{\mu}$  CC interactions occurring in the OPERA target, in total of 5489 events. When divided into two bins with a separation at 20 GeV, data produce results  $v_{\nu} - c = (2.17 \pm 0.83) \times 10^{-5}$  for low energy events with an average energy  $\langle E \rangle = 13.9$  GeV, and  $v_{\nu} - c = (2.74 \pm 0.80) \times 10^{-5}$  for high energy events with an average energy  $\langle E \rangle = 42.9$  GeV.

TABLE I: Data used for fitting from Fermilab [3, 4], MINOS [5], and OPERA [1].

Fermilab	Energy (GeV)	32	44	59	69	90	120	170	195
	$v_{\nu} - c$ ( $10^{-5}$ )	$-2_{-3}^{+2}$	$2 \pm 7$	$-1_{-3}^{+2}$	$-1_{-3}^{+2}$	$1_{-4}^{+3}$	$1 \pm 7$	$1_{-3}^{+2}$	$6_{-4}^{+3}$
MINOS	Energy (GeV)	3							
	$v_{\nu} - c$ ( $10^{-5}$ )	$5.1 \pm 2.9$							
OPERA	Energy (GeV)	13.9	42.9						
	$v_{\nu} - c$ ( $10^{-5}$ )	$2.17 \pm 0.83$	$2.74 \pm 0.80$						

Within our data preparation, there might be some unknown bias from MINOS data and OPERA data, but the bias is assumed to be small.

Now we look back to our dispersion relation Eq. (9), in which there are totally 20 unknown parameters since  $\mu$  and  $\nu$  goes from 0 to 3. Fortunately the number is largely reduced when certain symmetries are required in the model. For instance, if we want the theory to satisfy rotational invariance, non-diagonal entries in  $c^{\mu\nu}$  and  $a^i$  ( $i = 1, 2, 3$ ) vanish and there are only three nonzero parameters  $a^0$ ,  $c^{00}$  and  $c^{ii} = d$  ( $i = 1, 2, 3$ ). Moreover, we can require the CPT invariance to be an accurate symmetry thus get rid of the CPT-odd terms. Below we analyze the data in two simple cases.

**Case 1: All Lorentz violation coefficients but  $c^{00}$  vanish.**

In such a case the model is obviously invariant under CPT transformation and rotations. The dispersion relation reduces to

$$E = |\vec{p}| - \frac{1}{|\vec{p}|} c^{00} E^2. \quad (10)$$

By using the definition of the group velocity  $v \equiv \frac{dE}{d|\vec{p}|}$  one can easily get the velocity for neutrinos as

$$v_\nu = \frac{\sqrt{4c^{00} + 1} - 1}{2c^{00}}, \quad (11)$$

which is a constant. This is easy to understand since all  $c^{\mu\nu}$ s are dimensionless hence  $E$  is proportional to  $|\vec{p}|$ . So the energy dependence of the velocity would not be changed if we include  $c^{ii} = d \neq 0$  in the dispersion relation Eq. (10). The fit result is illustrated in Fig. (1) and the Lorentz violation coefficient  $c^{00}$  is constrained as  $(-1.80 \pm 0.48) \times 10^{-5}$ .

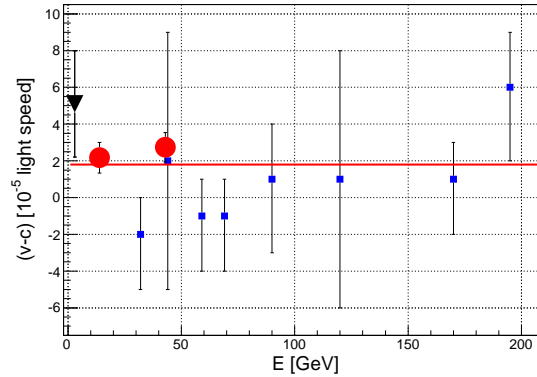


FIG. 1: The fit of a LV-modified velocity-energy relation,  $v_\nu = \frac{\sqrt{4c^{00}+1}-1}{2c^{00}}$ , to the moun neutrino velocity measurements from Fermilab [3, 4] (squares), MINOS [5] (triangles), and OPERA [1] (circles).

**Case 2:  $a^0 \neq 0$ ,  $c^{00} \neq 0$  while others vanish.**

The CPT invariance is spoiled by a nonzero  $a^0$  but the rotational symmetry still holds. The velocity-energy relation is given by

$$v_\nu = \frac{\sqrt{E(-4a^0 + 4c^{00}E + E)} - 2a^0 - (4c^{00} + 1)E}{\sqrt{E(-4a^0 + 4c^{00}E + E)} + 2a^0 + 4c^{00}E + E}. \quad (12)$$

The fit result is illustrated in Fig. (2) and the Lorentz violation coefficients are constrained as  $c^{00} = (-1.26 \pm 0.63) \times 10^{-5}$  and  $a^0 = (-5.89 \pm 4.42) \times 10^{-5}$  GeV. Despite the large uncertainties of  $a^0$ , we find in Fig. (2) that the contribution from CPT-odd terms becomes important in the low energy area. When energy approaches to above 50 GeV, the total shift of the neutrino speed with respect to light speed due to CPT-even terms dominates.

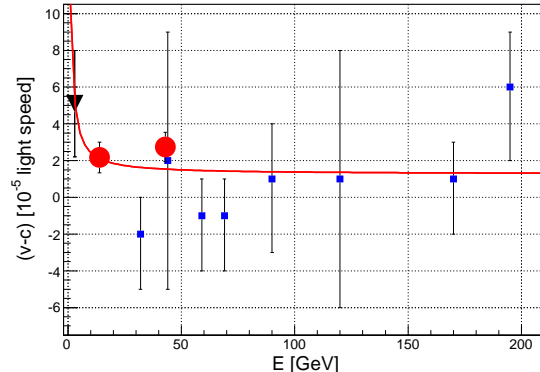


FIG. 2: The fit of a LV-modified velocity-energy relation,

$$v_\nu = \frac{\sqrt{E(-4a^0 + 4c^{00}E + E) - 2a^0 - (4c^{00} + 1)E}}{\sqrt{E(-4a^0 + 4c^{00}E + E) + 2a^0 + 4c^{00}E + E}},$$
 to the muon neutrino velocity measurements from Fermilab [3, 4] (squares), MINOS [5] (triangles), and OPERA [1] (circles).

There are many other choices of parameters and the fit results maybe quite distinctive. From the dimensional analysis one can find approximately that, CPT-even terms shift the velocity as a whole, and CPT-odd terms modify the relation between energy and velocity.

#### IV. SUPERNOVA 1987A

Besides the long baseline experiment, there are also measurable phenomenologies of superluminal neutrinos in astrophysics. For instance, supernova explosion (SNe) is an extremely luminous event, which causes a burst of radiation that outshines an entire galaxy. The radiation includes photons in a board range of spectrum, as well as neutrinos. Actually, most energy of a SNe is released in the form of neutrinos, however, due to the weak interactions of neutrinos with matters, only one event is observed with neutrino emissions on 23 February 1987, 7:35:35 UT ( $\pm 1$  min) — the Supernova 1987A in the Large Magellanic Cloud [30, 31], which is optically observed on 24 February 1987. More than ten neutrinos were recorded with a directional coincidence within the location of supernova explosion, several hours before the optical lights are observed. Because of weak interactions, neutrinos leak out of the dense environment produced by the stellar collapse before the optical depth of photons becomes  $< 1$ . Hence an early-arrival of neutrinos is expected. The journey of propagation of photons and neutrinos are of astrophysical distance ( $\sim 51.4$  kpc), hence it provides a unique opportunity to measure the speed of neutrinos to be within the light speed with a precision  $\sim 2 \times 10^{-9}$  [32].

Interestingly, while the OPERA results seem to be in remarkable consistence with other terrestrial muon neutrino velocity measurements, they contradict with the Supernova 1987A neutrino observation severely. The two toy models

in the previous section obviously can not describe this contradiction. One way out is taking more non-zero Lorentz coefficients into account, which will lead to very complex velocity-energy dependence and spoil some global symmetries. For instance, non-diagonal entries of  $c^{\mu\nu}$  will violate the rotational invariance. Moreover, we will need much more data to test such models because of the large parameter space.

Another remedy is the observation that actually the species of supernova neutrinos are different from those of terrestrial neutrinos — the former being electron neutrinos (and/or electron anti-neutrinos), while the measured neutrinos we are discussing are muon neutrinos. We suspect a family hierarchy should be responsible for the observed different velocities [24, 33]. In the dispersion relations, Lorentz violation coefficients of different flavors are generally different, hence if there exist family hierarchies of these parameters, the different propagation behaviors of Supernova 1987A neutrinos and terrestrial muon neutrinos can be understood.

It was argued by Cohen and Glashow [34] that the energy-losing process  $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$  becomes kinematically allowed when the Lorentz violation of OPERA muon neutrinos is of order  $10^{-5}$ , thus one would not see the muon neutrinos at the LNGS, where the OPERA experiment was performed. Bi *et al.* also argued that the Lorentz violation of muon neutrinos of order  $10^{-5}$  will forbid kinematically the production process of muon neutrinos  $\pi \rightarrow \mu + \nu_\mu$  for muon neutrinos with energy larger than about 5 GeV [35]. This kind of arguments has been recognized as a refutation for the rationality of the OPERA experiment. However, there have been a number of discussions [36–38], indicating that such an argument is not valid in general. The derived dispersion relation in the field theory frameworks could be covariant with the momentum of the muon neutrino and thus can avoid the Cherenkov-like radiations [2, 37]. The framework of SME has the potential to accommodate both the superluminality of neutrinos without the analogues Cherenkov radiation. This work might be considered as a first estimate of the magnitudes of the LV parameters in SME to fit the OPERA data, and more investigations are still needed from both experimental and theoretical aspects.

## V. CONCLUSION

The OPERA group measured the difference between the velocity of muon neutrinos to that of light, and reported a  $6\sigma$  significant indication that the muon neutrinos might travel with a speed slightly larger than that of light, which obviously contradicts with the most basic hypothesis underlying modern physics. Though unknown systematical errors can exist potentially, it still seems extremely worthy to look into possible theoretical reasons behind the observations. Lorentz-violating-induced modified dispersion relation appears to be the most robust possibility. With modified dispersion relation, the velocity of muon neutrinos can depend on their energies.

Within the framework of Minimal Standard Model Extension, we get the modified dispersion relation. From the dispersion relation Eq. (9), energy dependence of neutrino velocity is deduced in two rotational invariant models. We combine the muon neutrino velocity measurements from Fermilab, MINOS, and OPERA, to look into possible energy-dependence of neutrino velocity. The fit results imply that a constant shift from the speed of light, with a magnitude of order  $10^{-5}c$  is the contribution of CPT-even terms, while the energy dependence is due to CPT-odd terms.

We also explain the apparent conflicts between the Supernova 1987A neutrino observation and the muon neutrino velocity measurements. We point out that such a contradiction maybe solved by adding more parameters at the cost of symmetry breaking, or the contradiction maybe ascribe to family hierarchy of the Lorentz violation coefficients.

Either way, more data are needed to test the models.

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### Note Added

There is a piece of news that the OPERA collaboration has identified two possible effects that could have an influence on its neutrino timing measurement. The first possible effect concerns an oscillator used to provide the time stamps for GPS synchronizations, and the second concerns the optical fibre connector that brings the external GPS signal to the OPERA master clock. The two effects could have led to an underestimate of the flight time of the neutrinos, and a re-measurement of the neutrino speed by the OPERA collaboration will be done in the near future. If this is the reason for the earlier arrival time of neutrinos at the OPERA detector, the fitting result of this paper will need some modification and updating. Within the framework of SME, neutrinos can either be superluminal or subluminal depending on the sign of the parameters, so we still need more data to make a conclusion.

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